Parametric Value at Risk
Value at Risk (VaR) Definition

- VaR is defined as the potential change in market value over a 1-day period at the 99% confidence level due to changes in risk factors.
- The maximum likely loss on a portfolio for a given probability
Value at Risk Pros & Cons

**Pros**
- Regulatory measurement for market risk
- Objective assessment
- Intuition and clear interpretation
- Consistent and flexible measurement

**Cons**
- Doesn’t measure risk beyond the confidence level: tail risk
- Non sub-additive
Parametric VaR

Three Value at Risk Approaches

- Parametric Value at Risk
- Historical Value at Risk
- Monte Carlo Value at Risk
Parametric VaR

Parametric Value at Risk

- **Assumption**
  - Asset returns follow normal distribution

- **Pros**
  - Fast and simple calculation
  - Intuitive

- **Cons**
  - Poor accuracy for non-linear products
  - Second order approximation
  - Hard to incorporate stress test
Parametric VaR

**Parametric Value at Risk Methodology**

- Assuming an asset return/value change follows normal distribution, the quantile of 99% confidence level is $2.326\sigma$ where $\sigma$ is standard derivation.

- If absolute return $X_1 - X_0$ is normally distributed, the 99% worse change of $X$ is $X_1 - X_0 = 2.326\sigma$.

- The VaR is given by $\text{VaR} = \frac{\partial F}{\partial X} \Delta X = \frac{\partial F}{\partial X} \times 2.326 \times \sigma$ where $\frac{\partial F}{\partial X}$ is the delta.

- Similarly for a relative return $\frac{X_1 - X_0}{X_0}$, the VaR can be expressed as:

  $$\text{VaR} = \frac{\partial F}{\partial X} \Delta X = \frac{\partial F}{\partial X} (X_1 - X_0) = \frac{\partial F}{\partial X} \times X_0 \times 2.326 \sigma$$
Parametric Value at Risk Implementation

- For each asset/instrument/riskFactor, calibrate volatility $\sigma_i$ based on daily return.
- For each risk factor pair, calibrate correlation $\rho_{ij}$.
- Calculate the variance of a portfolio value change:

$$V_p^2 = \left| \Delta(P_1)\sigma_1 \quad \Delta(P_n)\sigma_n \right| \begin{bmatrix} \rho_{11} & \cdots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & \rho_{nn} \end{bmatrix} \begin{bmatrix} \Delta(P_1)\sigma_1 \\ \vdots \\ \Delta(P_n)\sigma_n \end{bmatrix}$$

- The portfolio VaR is $2.326 \sqrt{V_p^2}$. 
Parametric Value at Risk Implementation

- Calculate VaR values by multiplying the net PV01 values for each rating by the credit spread volatility factors for that rating. The credit spread volatility factor is the 1-day movement at the 99% confidence level based on the 2-year credit spread for a particular rating band.
Reference:
https://finpricing.com/aboutus.html